Conservation, Transitivity and Class Inclusion of Number

Tohoku Psychologica Folia
1979, 38(1–4), 8–17

CONSERVATION, TRANSITIVITY, AND CLASS INCLUSION OF NUMBER*

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This is a study of cognitive development in the children (4–6 yrs.) which the following two problems concern: (A) The developmental relationship among these cognitive skills of number: transitivity of inequality (*IT*), of equality (*ET*), conservation of equivalence (*EC*), of identity (*IC*), and class inclusion (*CI*). (B) The relationship between these cognitive skills of number and the size of number of items. Results obtained revealed that the order of task difficulty which also suggested the developmental order was: CI (the most difficult task)→*IC*↔*EC*→*ET*↔*IT*, and that the effect of the size of number of items arrived at the significant level only in *ET* and *IT*. The results were discussed in connection with Elkind's (1967) analysis of conservation task.

Problem

There remains an issue of the developmental relationship between the identity conservation (IC) and the equivalence conservation (EC) as one of the unsolved issues in conservation (C) study. This issue was originally raised by Elkind (1967). He analyzed the Piagetian conservation task and represented it like the following by means of symbols: to; S=V, t_1; V=V′ t_2; S?V′. Let us explain the symbols: "t" means time and "S" and "V" standard and variable stimulus presented to the subjects (*S*s). Namely they mean the following: The two stimuli S and V which are equivalent perceptually and conceptually are presented to the *S* (t_0). And then, the V is perceptually transformed into the V′, and the *S* is asked to judge whether the two (V and V′) are equivalent conceptually or not (t_1). Finally the experimenter (E) asks him (or her) to judge whether the two stimuli S and V′ are still equivalent conceptually or not (t_2).

In his analysis, Elkind noticed that Piaget seemed to be explaining how the subject arrives at the equality of S and V′ when, in fact, he is talking about the equality of V and V′. That is to say, Piagetian task requires the subject to judge the equality of S and V, and S and V′, but it is the equality of V and V′ that the E infers. It was necessary to arrange the task in its proper shape. In his analysis, the conservation task which aimed at knowing the *S*'s appreciation of the equality of V and V′, was categorized into an identity conservation task (IC). And the other conservation task which aimed at knowing the *S*s appreciation of the equality between two equivalent relations S=V and S=V, was into an equivalence conservation task (EC). From this

* The author wishes to thank Mr. Tomoaki Adachi for his warm assistance.

point, Elkind concluded that the conservation of equivalence reguires the utilization of immediate past experience in the form of a deductive argument in addition to the attainment of identity conservation.

It is logically true that the conservation of equivalence involves both the comprehension of transitivity (T) and the conservation of identity. Therefore, the following prediction is possible: The conservation of equivalence will not be attained prior to the conservation of identity and the comprehension of transitivity.

If this is true, the following hypotheses are possible: Hypothesis I; They are attained simultaneously in time. Hypothesis II; They are attained in a certain invariant order.

In studies on the developmental relationship between IC and EC hypothesis I, which is directly predicted by Piaget's theory, was supported by Murray (1970), Moynahan and Glick (1972), and Koshinsky and Hall (1973). On the other hand, hypothesis II was also supported by Schwartz and Scholnick (1970), Papalia and Hooper (1971), Elkind and Schoenfeld (1972), and Litrownik et al. (1978). The causes of this discrepancy among their findings seems to be attributed to the difference of their experimental procedures. Brainerd and Hooper's (1975) analysis of the discrepancy concluded that those findings which supported hypothesis I were able to be attributed to measurement error (response criterion) and/or sampling error (the *S*s' age), that is, the employment of "response and explanation" criteria and/or the employment of the older *S*s was likely to induce a simultaneous attainment of IC and EC.

In this experiment, the *S*s who were not so young and not so old were employed and both "response only" and "response and explanation" criteria were employed. In this condition, we tried to find the developmental relationship between IC and EC of number.

Elkind's analysis of Piagetian conservation task described above also predicted that the developmental relationship between T and EC will be like the following: Conservation of equivalence will not be attained prior to the comprehension of transitivity. That is to say, they will be attained simultaneously in time or transitivity will be attained prior to conservation of equivalence. We tried to decide which is true by employing the same materials both in T and in EC. A finding of this study seems to be worth trying in the point that there were few studies which dealt with a number both in transitivity task and in conservation task in the past.

Brainerd (1974) reported that the order of the degree of the improvement after training was transitivity—conservation—class inclusion (CI). In his experiment, however, the same materials were not employed, that is, he employed strings for length conservation, sticks for length transitivity, and line drawings for class inclusion of length. As Anderson and Cuneo (1978) suggested that materials employed in conservation task could be a variable which influenced the *S*s' performances, it should not be desirable to employ the different materials for the different tasks to compare the performances among those tasks. In this experiment, we employed the same materials for the tasks.

The relationships between the size of the number of items and these cognitive skills of the number described above were the second major problem in this experiment. Especially, our concern was of conservation.

Recently, Cowan (1979) reported that the *S*s' (5 yrs.) performance in IC was improved with the decrement of the number of items. In detail, the performance on the 2-item version was significantly superior to the performance on the 5-item version and also to the performance on the 15-item version ($p<.01$, respectively). The performance on the 5-item version was significantly superior to the performance on the 5-item version ($p<.02$). The same results were reported by Winer (1974) and Young and McPherson (1976) also in EC task. These are interpreted as evidences to show that those children who succeed in appreciating number invariance in the small number of items do not necessrily appreciate invariance of number in the large number of items. In this experiment, we tried to reconfirm these results and in the same time tried to obtain an evidence concerning the size effect of the number of items in T task and in CI task. We have not seen such an evidence in the literature.

METHOD

Subjects: The *S*s were 96 kindergarten children in Sendai city (medium sized city in Japan). There were 43 boys and 53 girls. Those children who failed in demonstrating understanding of the concepts of "more", "less", and "the same number" had been omitted from this experiment.

Design: The *S*s were devided into 12 groups according to a 3 (age: 4 yrs., 5 yrs., 6 yrs.) ×4 (the number of items: 3, 5, 9, 11) factorial design. Their mean age in each age group was 4 yrs. and 9 mo., with an SD of 2.4 mo., 5 yrs. and 7 mo., with an SD of 3.2 mo., and 6 yrs. and 4 mo., with an SD of 2.6 mo. The *S*s were administered all five kinds of task, that is to say, identity conservation (IC), equivalence conservation (EC), transitivity of equality (ET), transitivity of inequality (IT), and class inclusion (CI). Each task was consisted of one trial. The order of task presentation and the order of questioning ("same - different" or "different - same") were random.

Materials. Forty-four caramels (22 white and 22 brown), 3 boards (blue; 7.9cm× 89 cm) with 25 rectangular boxes (2.3 cm×2.7 cm), and a sheet of paper (25.6cm× 36.2 cm) were employed. The distance between two boxes was .5 cm.

Procedure: Identity Conservation (IC): A row of caramels consisted of 3, 5, 9, or 11 items were presented to *S* and he (or she) was told, "Here are caramels. Now, look! I am going to expand the interval between these caramels." After the interval was expanded, the *S* was asked ,"Now, are there the same number of caramels as before, or a different number of caramels?" If the *S* answered "same", he (or she) was immediately asked the explanation of his (or her) response ("Why did you think so ?"). If the *S* answered "different", he (or she) was asked the next question, "Which has more caramels, the expanded row or the unexpanded one ?" After his (or her) response, he (or she) was asked for the explanation of his (or her) response (Fig. 1. (a)).

Equivalence Conservation (EC): The two rows of caramels which were the same in number and in position on the board, but were different in color were presented to *S*. The distance between two boards was about 10 cm. The *S* was asked if the number of white caramels was the same as the number of brown ones or not. If the *S* answered "not the same," then the *E* demonstrated the equality of the two rows of caramels in number showing one-to-one correspondence. There was no child who did not understand the equality of these caramels even after the demonstration. After the *S* understood the equality the *E* expanded the row of caramels on the board that was in a more distant position, and asked the *S*, "Is the number of white caramels the same as the number of brown ones, or the number of white ones different from the number of brown ones?" The rest is omitted, for it is the same as in IC (Fig. 1. (b)).

Equality Transitivity (ET): In this task, we employed three boards. This task involved three judgements of equality: A=B and B=C, therefore A=C (the number of caramels on the board A is the same as that on the board B and the number of caramels on the board B is the same as that on the board C, therefore the number of caramels on the board A is the same as that on the board C.)

First, three rows of caramels were presented to the *S*. A row of caramels on the board C was covered with a sheet of paper. Each caramel on the board A (the nearest to the *S*) was in the standard position (Fig. 1(C)) Each caramel, however, on the board B was shfited by just one box to the right hand side from the standard position, and each on the board C to the left hand side from the standard position. And then, board B was so shifted toward the board A that each caramel on the board A corresponded to each on the board B, and the *E* asked the *S*, "Is the number of these caramels (pointing to the row of caramels on the board A) the same as the number of these caramels (pointing to the row of caramels on the board B) or not ?" If the *S* answered "not the same," then the *E* demonstrated the equality of the two rows of caramels in number showing one-to-one correspondence. There was no child who did not understand the equality of the number of these caramels even after the demonstration. After the *S* understood the equality, the board B was returned to the place where it had been located. Next, the row of caramels on the board A was covered with the sheet of paper which had covered the row of caramels on the board C. And then, the board B was so shifted toward the board C that each caramel on the board B corresponded to each on the board C, and the *E* asked the *S* the same question described above. After the *S* understood the equality of the number of caramels on the board B and on the board C, the board B was returned to its former position. Finally, the *S* was told, "Is the number of these caramels (pointing to the board A) the same as the number of these caramels (pointing to the board C), or the number of these caramels (pointing to the board A) different from the number of these caramels (pointing to the board C) ?" The *S* was also asked for an explanation of his (or her) response. Inequality Transitivity (IT): In this task, the procedure was the same as that in ET except that the caramels on the board C in this task was fewer than in ET by one (Fig. 1 (d)). In both the two tasks, the color of caramels on the board A was the same as that on the board C. In color, the caramels on the board B differed from both the caramels on the board A and on the board C.

Class Inclusion (CI): In this task, only one board was employed. A row of two colors of caramels, that is, one was white and the other was brown, and one of the two was fewer than the other by one caramel, was presented to the *S*. The *S* was told, "Here are caramels. These are white and these are brown (pointing to the white caramels and the brown ones). Which is more, the white caramels or the brown ones?" If the *S*'s response was correct, and if the brown caramels were more than white ones, he (or she) was asked, "Which is more, all the caramels or the brown ones (tracing all the caramels and brown ones by finger)?" After his (or her) responding, the E asked him (or her) for an explanation of his (or her) response. (Fig. 1(e))

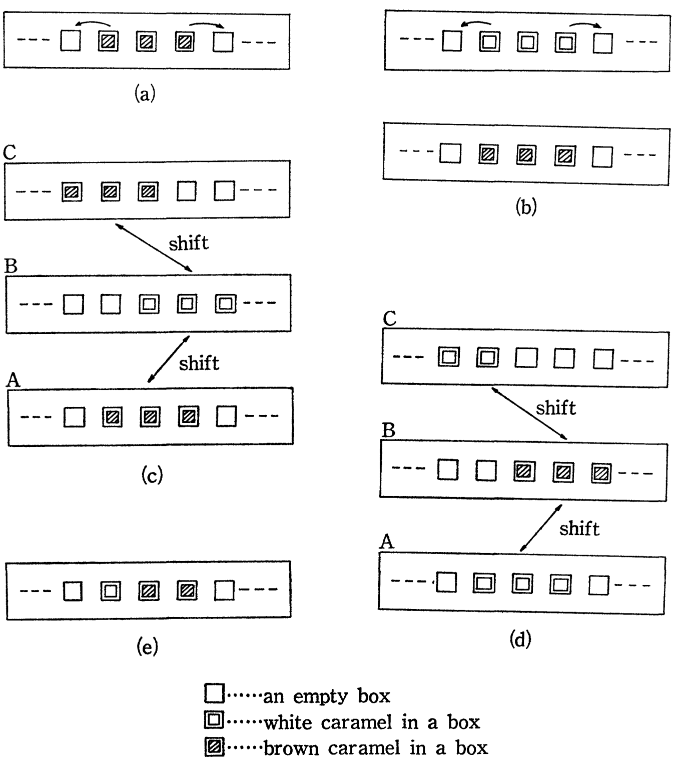

Fig. 1. Chart of procedure in each task in the case where 3-item version was employed.

Results

Scoring: For the statistical purposes, to each correct judgement followed by a reasonable explanation of this judgement was allotted "1" point and to each incorect judgement or correct judgement followed by no reasonable explanation was allotted "0" point in the case where a "response and explanation (R-E)" criterion was employed. The criteria of reasonable explanations were: correct counting for each task; identity or two kinds of reversibility for IC and EC; correct utilization of mediate term for ET and IT; part-whole comprehension for CI.

On the other hand, to each correct judgement was assigned "1" point and to each incorrect one "0" in the case in which a "response only (R-O)" criterion was employed.

Analysis of data: In the case in which R-O criterion was employed: The obtained data were analyzed with an analysis of variance method. We could not obtain any significant interaction but as for the main effects we obtained the same conclusion as in the case where R-E criterion was employed. We showed these results, comparing with the results in the case in which R-E criterion was used.

In the case where R-E criterion was employed; The obtained data were analyzed with an analysis of variance method. Results shown at table 1 revealed that each main effect except the effect of the number of items arrived at the significant level ($p<.01$) and that the age $\times$ task interaction and the task $\times$ number of items interaction were significant ($p<.05$ and $p<.01$).

Since the age difference effect arrived at the significant level, the comparison between ages was undertaken. It was found that the *S*s' performance improved with the increment of their ages, but that there was no significant performance difference only between the four-year-old children and the five-year-old children (Table 2). The method employed for comparing the *S*s' performance between two age groups was the modified least significant difference method.

Table 1. Results of ANOVA (F ratio)

Factors \ Criteria	R–E	R–O
Age (A)	10.802**	7.401**
Item (I)	1.811	.321
(A)×(I)	1.511	.881
Suject (S)	3.518**	1.835**
Task (T)	20.670**	30.318**
(A)×(T)	2.241*	1.294
(I)×(T)	3.393**	1.612
(A)×(I)×(T)	.893	.712

* p<.05 ** p<.01

To analyze the age $\times$ task interaction, the age difference in each task was computed: CI; $F(2, 95)=.1498$, IC; $F(2, 95)=2.696$, EC; $F(2, 95)=7.856$ ($p<.01$), ET; $F(2, 95)=5.045$ ($p<.01$), and IT; $F(2, 95)=9.614$ ($p<.01$). Only CI and IC did not yield a significant age difference effect.

The comparison between tasks was undertaken, for we obtained the significant task difference effect. There was no significant performance difference only between EC and IC and between ET and IT (Table 3). Therefore we can decide the task difficulty order like T→C→CI (the most difficult).

An analysis of the task×age interaction evidenced that the task difference effect arrived at the significant level in each age group (Fig. 2). To say more exactly, it was found that the task difference effect in the four-year-old children was weaker ($F(4, 124)=3.248$, $p<.05$) than in the five-year-old children ($F(4, 124)=14.309$, $p<.01$).

The task×number of items interaction was also computed. The task difference effect was significant in each number of items (3-item version; $F(4, 92)=19.094$, $p<.01$, 5-item version; $F(4, 92)=3.581$, $p<.01$, 9-item version; $F(4, 92)=3.286$, $p<.05$, 11-item version; $F(4, 92)=2.537$, $p<.05$). Weaker effects in 9 and 11 item versions were observed. The effect of number of items arrived at the significant level only in ET and IT ($F(3, 84)=3.746$, $p<.05$; $F(3, 84)=7.696$, $p<.01$).

Since a stronger individual difference effect was observed, we analyzed the frequency data of the *S*s' pass-fail patterns for each task. Eighteen patterns were observed in all. An analysis of variance of these data revealed significant effects of pattern difference ($F(17, 102)=16.884$, $p<.01$), the pattern×age interaction ($F(34, 102)=3.942$, $p<.01$) and the pattern×number of items interaction ($F(51, 102)=1.942$, $p<.01$). Though pattern difference in each age group and in each number of items group was significant (4 yrs.; $F(17, 54)=15.647$, $p<.01$, 5 yrs; $F(17, 54)=4.615$, $p<.01$, 6yrs.; $F(17, 54)=2.759$, $p<.01$ and 3–item version; $F(17, 36)=3.812$, $p<.01$, 5-item version; $F(17,36)=4.160$, $p<.01$, 9-item version; $F(17, 36)=2.118$, $p<.05$, 11-item version; $F(17, 36)=4.941$, $p<.01$), the age difference effect in each pattern was not significant except in two patterns, and the number of items effect in each pattern was not significant. Two patterns varied in emergence frequency with the increment of the age were a decreased pattern "O(CI)–O(IC)–O(EC)–O(ET)–O(IT)" and an increased pattern

Table 2. Between ages comparison

Criteria	R–E		R–O	
Ages	5	6	5	6
4	21	50**	14	38**
5		29*		24*

* p<.05 ** p<.01

Table 3. Between tasks comparison

Criteria	R–E				R–O			
Tasks	IC	EC	ET	IT	IC	EC	ET	IT
CI	19**	25**	35**	37**	38**	37**	50**	58**
IC		6	16**	18**		−1	12*	20**
EC			10*	12**			13*	21**
ET				2				8

* p<.05 ** p<.01

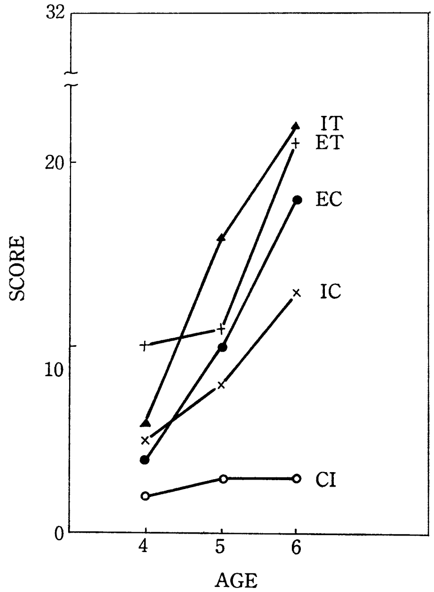

Fig. 2. Scores of 4-year-old, 5-year-old, and 6-year-old children in each task.

"0–1–1–1–1". Another one which emerged more frequently than others but did not significantly vary with the increment of the age was pattern "0–0–0–1–1".

Discussion

The result showing that there was no significant difference in their performances between the four-year-old children and the five-year-old children, but was a significant difference between the five-year-old children and the six-old-children appears to suggest that an age of the six-year-old is an important age on the course of children's cognitive development. We have, however, no evidence to explain why an age of six-year-old is important.

In task difficulty we obtained the relationship T→C→CI. It also seems to reflect a developmental relationship among the tasks, for an analysis of the *S*s' pass-fail patterns among the tasks suggested that a pattern "0–0–0–1–1" which corresponded to "CI–IC–EC–ET–IT" emerged earlier than a pattern "0–1–1–1–1". Only the pattern "0–0–0–1–1" of the three patterns which emerged more frequently than the others did not reveal the significant age difference eeffect, while the other two "0–1–1–1–1" and "0–0–0–0–0" revealed it. We should remember here that the two patterns "0–0–0–1–1" and "0–1–1–1–1" emerged with almost the same frequency. It was only the pattern "0–1–1–1–1" that arrived at the significant level of the age difference effect, although the emergence frequencies of both the patterns "0–0–0–1–1" and "0–1–1–1–1" tended to increase with the increment of the age. These appear to suggest that T was attained earlier than C. Similarly it appears to be suggested that C was attained earlier than CI. Thus, we can conclude that the developmental relationship among the tasks is T→C→CI. This conclusion is congruent with Brainerd's (1974) finding.

The prediction that "EC will not be attained prior to IC or ET", which was induced from Elkind's (1967) analysis of Piagetian conservation task was empirically confirmed, for we obtained the finding which suggested that T was attained prior to EC and that IC and EC were attained simultaneously in time. Although we have not any sufficient evidence to explain why T was attained prior to EC or why IC and EC were attained simultaneously in time, we can present a few possibilities which may explain these results.

Brainerd and Hooper (1975) explained the results of "IC↔EC" in many studies with a measurement error (the employment of "response and explanation" criterion) and/or with a sampling error (the employment of the older, more than six-year-old, children). Their explanation, however, seems not to be able to be generalized to this experiment, for we obtained the same "IC↔EC" both by "response only" criterion and by "response and explanation" criterion employing the children who were not too young and too old.

We should be reminded of Gelman's (1972) indication that in conservation task not only "logical ability" but also "control of attention", "understanding of semantics", and "estimation skills" are measured, for we observed that many children explained their judgements based on "counting" in EC and IC. Concretely, 46.2 per cent and 43.8 per cent of the children who succeeded in IC and in EC employed counting based explanations. The explanation it self based correct counting does not necessarily mean the attainment of appreciation of invariance of number, but it is possible to succeed in number conservation task by means of it. Therefore it is probable that we obtained an apparent relationship "IC↔EC" as a result when the relationship "IC→EC" exsisted in truth. In fact, Cowan (1979) succeeded in separating the successful performance attributed only to applying requantifying skills from the other successful performance by screening method in IC. Therefore, perhaps we had better postpone deciding whether IC is attained prior to EC or IC and EC are attianed simultaneously in time.

We observed the significant effect of the number of items only in ET and IT. And also we found that only the smallest number of items, 3 items version, differed in performance from others. Cowan (1979) reported that the effect of employing requantifying skills was found only on the small number of items version in IC task. Possibly a similar effect was observed only in T in this experiment. A good many children employed explanation based on counting in ET and IT, amounting to 88.2 per cent and 86.5 per cent of the children who successfully accomplished these tasks. T was a task in which *S*s find it apparently easier to apply counting skill than C. The result T→C may concern this fact.

The non-significant effect of the number of items in IC and EC suggests that those who can attain conservation in the smaller number of items version can attain it even in the larger number of items version at least in the number range 3–11. We have had, however, no clear-cut data which can explain this result as yet.

CI task seems to be difficult for our *S*s to reveal significant age difference and the difference of the number of items.

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(Received November 30, 1979)